AD-A032 847 LOUISIANA STATE UNIV BATON ROUGE DEPT OF CHEMICAL EN--ETC F/G 12/1
PROCESS IDENTIFICATION UTILIZING A SEQUENTIAL INSTRUMENTAL VARI--ETC(U)
1976 A T TOUCHSTONE, A B CORRIPIO AF-AFOSR-2580-74
UNCLASSIFIED AFOSR-TR-76-1171 NL

| OF |
AD
A032847

END
DATE
FILMED
1-77

3

AFOSR - TR - 76 - 1171

See Form 1473

PROCESS IDENTIFICATION UTILIZING A SEQUENTIAL

INSTRUMENTAL VARIABLE REGRESSION ALGORITHM

by A. T. Touchstone* and A. B. Corripio

Louisiana State University

Baton Rouge, Louisiana 70803

Abstract

Instrumental variable (IV) regression is applied to the estimation of the parameters of a difference equation model of a process subject to noise. The technique preserves the simplicity of least-squares estimation, and is shown to significantly reduce the bias on the parameter estimates caused by measurement noise. A second-order example is used to illustrate the performance of the IV estimator, and to study the selection of sample time and initialization parameters. Demonstration is given on the parameter tracking capability of the dynamic form of the algorithm. The dynamic algorithm is important for use in adaptive control.

Introduction

Today, with the aid of an on-line digital computer, it is possible to use an empirical model to represent a non-linear plant in a sequential manner. Specifically, a low-order linear model is selected to represent the dynamic system. By use of the proper estimation algorithm, the computer is able to produce updated parameter estimates for the empirical model at each sample instant. With this quasi-linearization procedure it is now feasible for the engineer to characterize the dynamics of his process sufficiently to be able to apply such advanced control techniques as the self-adaptive strategy.

This paper presents one method of process identification - namely, the instrumental-variable method. This method can be used to estimate model parameters from an array of linear algebraic equations involving these parameters, a set of observations, and a set of unobservable noise terms. The estimates are obtained by first multiplying the array by a rectangular matrix of "instruments" to yield a square invertible matrix which is then solved for the estimates.

Definition of the Problem

Stated simply, process identification is the problem of determining a description of the relationship between the input and output variables of the process. The fact that the unknown plant has been *selected for* study may suggest non-linear behavior, time-variable parameters, or simply a linear system operating in an extreme environment. Further, the plant may have a number of input and output variables in which case it is termed multivariable. It is important to make the distinction between the identification problem and the similar, but inverse, problem of output estimation. In estimation, the physical relationship between the input and output is given (known) and either a filtered or predictive estimate of the output is desired. The problem that is generally referred to as "optimal" requires a statistical description of all noise and disturbance processes introduced into the system.

More specifically, the following identification problem is considered. Given the discrete-data record of a sequence of inputs $\{u(t);\ t=1,2,\dots,K\}$ or $\{u(t)\}_K$ and the corresponding, disturbance-corrupted outputs $\{y(t)\}_K$ of a dynamical system, determine the parameters of a suitable linear model to be fitted to the data record. The number of observations, K, is larger than the number of parameters to be *estimated*.

To be of practical use in an adaptive control strategy, the problem as stated is too general and further restrictions are needed. Some major requirements of adaptive philosophy are: (1) that identification be performed "on-line" and in the presence of the system's normal operating signals and disturbances, using only the normal control inputs; (2) that the computation time for identification be reasonably short compared to the sampling time required to maintain good control; (3) that the computation time be short compared to the rate of variation of the process parameters; (4) that data storage be minimal. These requirements suggest a recursive technique which is capable of improving its estimates as more data is received and is able to detect parameter variation at the same time. The general approach is then a quasi-linearization procedure which yields a model of the plant at each sampling instant.

Specification of the Model

The first step of the identification problem is the specification of an algebraic structure between the input and output variables. This model is postulated a priori and stems from the underlying theory governing the particular dynamic system under investigation. Because most adaptive control and optimization problems are time-varying and/or nonlinear, convenience has replaced the classical frequency-domain analysis with time-domain methods. Of these, discrete-time methods are preferred when the data records are obtained by sampling the input and output signals at regular intervals. Since this is the manner in which

*Current address: Dow-Badische Chemical Co., Freeport, Texas.

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DDC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12 (7b).
Distribution is unlimited.
A. D. BLOSE
Technical Information Officer

the data are gathered by on-line digital computers, the most convenient model is in the form of difference equations.

With reference to the identification scheme sketched in Figure 1, the following difference equation can be used to represent the relationship between the single input and the single disturbance-free output, c_t[1]:

$$c_t = a_1 c_{t-1} + a_2 c_{t-2} + \dots + a_p c_{t-p} + b_1 u_{t-M-1} + b_2 u_{t-M-2} + \dots + b_q u_{t-M-q} \quad (1)$$

where M represents the process dead time in integer multiples of the sample interval, T, and p and q are the number of output and input terms respectively. It is generally accepted that, for process control applications, the model need not be of order higher than second ($p = 2$, $q = 2$).

Equation 1 can be expressed in compact vector (or matrix) notation as follows:

$$c_t = \varphi x_{t-1}; \quad t = 1,2,\dots,K \quad (2)$$

where

$$\varphi = [a_1 a_2 \dots a_p b_1 b_2 \dots b_q]$$

$$x_{t-1}^T = [c_{t-1} \dots c_{t-p} u_{t-M-1} \dots u_{t-M-q}]$$

If c_t could be measured without error and the model fit the process exactly, then only $(p + q)$ sets of data (observations) would be required to determine the model parameters exactly. That is $(p + q)$ observations would give $(p + q)$ equations with as many unknowns. In practice, however, there exist three major sources of uncertainty, namely:

(1) The inability of the model to describe the process exactly,

(2) The disturbances entering the process (w_d in Figure 1), and

(3) The presence of measurement noise, w_n.

To account for these uncertainties, equation 2 can be extended as follows:

$$y_t = \varphi x_{t-1} + w_t \quad (3)$$

where

$$x_{t-1}^T = [y_{t-1} \dots y_{t-p} u_{t-M-1} \dots u_{t-M-q}].$$

The term w_t represents more than just the simple combination of w_d and w_n. As the vector x of past input-output observations contains now the noise-corrupted measurements (y) rather than the true plant output (c), w_t would be correlated even if w_d and w_n are not.

The Structural Relationship

It would at first seem proper to estimate the model parameters of Equation 3 by application of either ordinary least-squares (LS) or generalized least-squares (GLS). The choice would appear to depend upon the particular assumptions made on the disturbance term. The danger implicit in this procedure has been pointed out by Young [2]. Equation 3 does not represent the simple regression model upon which the least-squares theory is based, but instead contains "errors in variables" within the x_t vector and is known as a "structural model" [3].

The elements of the observation vector x_t constitute what are called explanatory variables (independent variables) and application of LS requires that these variables be known exactly (deterministic) or at least be uncorrelated with the disturbance term *if the explanatory variables* are themselves random. That is, it must be assumed that

$$E[w_t x_t] = 0$$

for consistency, if not unbiasedness. Now, the vector x_t contains passed values of the input and output of the process. These passed values are known as "lagged" variables. The passed values of the input are lagged values of the independent variable and cause no difficulty for open-loop identification. Lagged values of the output (dependent) variable, on the other hand, represent a really troublesome obstacle - namely, that the *explanatory variables are correlated with the disturbance term*. Using the results of simple linear regression analysis in this case will result in estimates which are biased in finite samples to a degree dependent on the noise-to-signal ratio. If in addition, the disturbance term is autocorrelated, then the combination of lagged variables and autocorrelated disturbances result in the LS estimator being inconsistent [4]. Hence, in any application of LS theory, it must be assumed that the conditions for the validity of the results are satisfied.

Survey of Estimation Methods

In the last section, the problems encountered in the application of classical regression to the structural model of Equation 3 were discussed. The technique of GLS requires complete specification of the disturbance for direct estimation of the model parameters. An on-line GLS method of estimating the coefficients of a signal processing filter to approximate the disturbance is presented by Hastings-James and Sage [5].

There are two main estimation methods which can be utilized with the structural model. One is the instrumental variable (IV) approach. The other technique is the method of maximum likelihood estimates. This method, originally developed for off-line computation, carries fairly strong assumptions about the covariance matrix of noise - namely, all the random variables involved are assumed to be Gaussian. Moreover, initial guesses of the parameters must be made close to the true values in order to guarantee convergence.

Two recent papers [6,7] deal specifically with the comparisons of the performance and computational expense of all the important on-line identification methods. Isermann et al. [6] concluded that identification methods using the same a priori information of the process model results in about the same performance, if the "laws of good identification" are applied. They suggest that the key to selection of an identification method rest on such factors as the kind of input signal, the computational expense, the overall reliability and the necessary assumptions required.

Instrumental Variable Estimator

The difficulties encountered in obtaining an unbiased, minimum variance estimator are attributed to the correlation arising between w_t and x_{t-1} of the structural model of Equation 3. The instrumental variable (IV) approach to parameter estimation was first introduced in the early 1940's in an area of economics now known as econometrics [8]. The method was specifically designed to deal with the errors-in-variables problem, i.e., the errors associated with the explanatory variables in regression calculations [3,9]. The IV method is intended as a compromise between the range of techniques from largely deterministic procedures to sophisticated statistical methods based on the results of optimal estimation theory.

The first use of the IV approach in the field of process identification is generally attributed to Joseph et al. [10]. They suggested an IV procedure utilizing the input variable u_t as the instruments for identifying the parameters of a process described by a difference equation model. Andeen and Shipley [11] reported an essentially similar technique of the Joseph et al. method with an added prefilter to improve the accuracy of estimation in the presence of noise. Levadi [12] presented a purely analog IV method for identifying a linear dynamic process described by a differential equation model. Wong and Polak [13] showed that there exist optimal instrumental variables and they approximated the optimal variables with the calculated, undisturbed output of an auxiliary model. In turn, the parameter estimates were used to update the auxiliary model. Young [14] presented a hybrid development of Levadi's purely analog approach and introduced a time delay and a low pass filter before updating the auxiliary model to ensure that the model parameters were not correlated with w_t at the same instant and to smooth the estimates. In addition, Young presented the dynamic form of the IV algorithm and discussed its relationship to the Kalman filter equations. Rowe [15] extended the estimator to the multivariable case and included a spectral factorization technique for identifying part of the correlated disturbance.

The instrumental variable technique is best discussed in terms of the regression equations. Application of Equation 3 to the K sets of observations results in the following matrix equation:

$$Y_K = \varphi_K X_{K-1} + W_K \qquad (4)$$

where

$Y_K = [y_1 y_2 \dots y_K]$

$X_{K-1} = [x_0 x_1 \dots x_{K-1}]$ (a (p+q)xK matrix)

$W_K = [w_1 w_2 \dots w_K]$

This equation is now post multiplied by a transformation matrix, Z_K^T:

$$Y_K Z_{K-1}^T = \varphi_K X_{K-1} Z_{K-1}^T + W_K Z_{K-1}^T \qquad (5)$$

This transformation, or instrumental variable, matrix is chosen which satisfies

$$E[W_K Z_{K-1}^T] = 0 \qquad (6)$$

$$E[X_{K-1} Z_{K-1}^T] \quad \text{nonsingular}$$

The elements of Z_K^T are therefore chosen to be uncorrelated with the disturbance W_K, and at the same time, highly correlated with the regressors in X_{K-1}. Solving Equation 5, the IV estimate is given by

$$\hat{\varphi}_K = Y_K Z_{K-1}^T [X_{K-1} Z_{K-1}^T]^{-1} \qquad (7)$$

If the observation matrix, X_{K-1}^T, is used as the IV matrix, LS estimation results, thus becoming a special case of the IV approach. Therefore, by proper choice of an IV matrix, it is possible to eliminate the bias due to the combination of noise and lagged variable regressors while preserving the simplicity of least-squares estimation.

Unfortunately, the universal law of the conservation of "advantage" is valid in this instance. The law states that to gain an advantage in one area requires a proportional disadvantage be established in another area. In this instance, the elimination of asymptotic bias, i.e. consistency, is accompanied by a certain loss of efficiency in the statistical sense. IV estimators lack efficiency because they make use of only a limited amount of a priori information. But the lack of a priori disturbance statistics was the major reason for selecting the IV approach. So efficiency has not really been lost - the problem requirement is simply more demanding. And as might be expected, the greater the correlation between the instrumental variable, z_t, and the noise-free signals, c_t, the smaller the estimation variance. Finally, a small bias will remain due to finite sample lengths.

Since a major advantage of the IV method is the preservation of the simple least-squares structure, the recursive or sequential form of Equation 7 is desirable. The recursive relations are

$$\hat{\varphi}_{t+1} = \hat{\varphi}_t + (y_{t+1} - \hat{\varphi}_t x_t)[1 + z_t^T P_{t-1} x_t]^{-1} z_t^T P_{t-1} \qquad (8)$$

$$P_t = P_{t-1} - P_{t-1} x_t [1 + z_t^T P_{t-1} x_t]^{-1} z_t^T P_{t-1} \qquad (9)$$

where

$$P_{t-1} = [X_{t-1}Z^T_{t-1}]^{-1}$$

A diagram for the sequential IV estimator is shown in Figure 2. Wong and Polak [13] and Rowe [15] have pointed out, in regard to the selection of z_t, that the direct GLS estimator represents the optimal IV estimator. Referring to Figure 2, this implies the use of c_{t+1} as the instrumental variable which in turn implies the precognitive solution to the problem by requiring complete a priori information on the noise statistics. Utilization of the output of an auxiliary model, as indicated, represents a close approximation to c_{t+1}. A by-product of this procedure is, of course, filtered estimates of the system output.

Parameter Tracking

A decided advantage of the recursive relations of Equations 8 and 9 over their nonrecursive counterparts is that it is no longer necessary to store and perform calculations on large quantities of data. The data is reduced at each sample instant and is accumulated with past reduced data in the considerably compact weighting-matrix, P_K. Hence the P_K matrix represents a concise history of the observations. The weighting-matrix is therefore the key to parameter tracking.

Relations like Equations 8 and 9 represent finite-time averaging operations in which all data is weighted equally over the observation interval of K samples. An important assumption implicit in this operation is that the unknown parameters are constant over the observation set [2]. It then follows that in order to detect parameter variation, it is necessary to shorten the memory of the estimation procedure so that new observations are given more weight than outdated observations.

One approach to parameter tracking is obtained by weighting the data exponentially into the past to gradually remove information as it becomes obsolete. Examination of Equation 9 for the finite-time averaging case will show that P_K is strictly a decreasing function of time. After a large number of samples (large K) have been collected, P_K may diminish to the extent that new data no longer influences the estimates. That is, due to the averaging process, individual samples have less effect on the estimates as the number of samples, K, increases. The physical effect of exponential weighting the data is to set a lower bound on P_K so that new data continues to influence the estimates via Equation 8 [16].

It is important to point out that this approach has the disadvantage that the effects of noise will also be detected and used to modify the estimates. It will therefore be necessary to assume that the parameter variations are larger than the residual fluctuations due to noise.

An alternative to exponential weighting the data for parameter tracking is to describe the parametric variation by a suitable stochastic model. Lee [17] points out that the resulting algorithm is not optimal and its only justification is that "it works". It is interesting to note that this approach results in a prediction-correction algorithm which is a special form of the Kalman filter equations [18,19].

When the structure of the stochastic parameter variation model is not known, a simplified form may be given by:

$$\varphi_t = \varphi_{t-1} + q_{t-1} \qquad (10)$$

The dynamic IV algorithm is correspondingly simplified to

$$\hat{\varphi}_{t+1} = \hat{\varphi}_t + (y_{t+1} - \hat{\varphi}_t x_t)[1 + z_t^T P_{t/t-1} x_t]^{-1} z_t^T P_{t/t-1} \qquad (11)$$

$$P_{t/t-1} = P_{t-1} + D \qquad (12)$$

$$P_t = P_{t/t-1} - P_{t/t-1} x_t [1 + z_t^T P_{t/t-1} x_t]^{-1} z_t^T P_{t/t-1} \qquad (13)$$

where $P_{t/t-1}$ is the a priori update of P at time t, based on observations up to and including y_{t-1}. The only difference between this dynamic algorithm and the static scheme of Equations 8 and 9 is the addition of Equation 12. The D matrix simply adds the necessary lower bound to P so that the algorithm can track parameter variations.

Although this is the same result obtained by exponential weighting the data, the dynamic approach has more inherent flexibility and is more easily implemented. Since D is a matrix it is possible to limit individual elements to different degrees. For example, if certain parameters are known to be time-invariant, then the corresponding elements of D would be zero. Further, the model approach allows any a priori information that is available to be used in the estimation algorithm. Young [14] illustrates this last point by including in the estimation additional measured data obtained from the process.

Computational Results

The results presented here are based on the second-order plant described in Table I. The system was forced with discrete white noise, i.e., a pseudo-random-binary-sequence (PRBS), with a clock interval equal to the sample interval T. For details on this type of input signal see Eveleigh [20] or Davies [21]. The PRBS starter sequence was not varied between runs so that the input would be reproducible.

TABLE I
Second Order Plant

The Second-Order Linear Process

$$2\frac{d^2c}{dt^2} + 3\frac{dc}{dt} + c = u(t)$$

results in the following difference equation parameters:

$a_1 = 0.97441 \qquad b_1 = 0.15482$

$a_2 = -0.22313 \qquad b_2 = 0.09390$

for a sample interval T = 1. The time constants and gain are

$\tau_1 = 2 \qquad \tau_2 = 1 \qquad k = 1$

Measurement noise was simulated as discrete white noise, v(t), generated by a pseudo-random number generator routine. The routine generated uniformly distributed real numbers which were then transformed into a normally distributed random number sequence with a given mean and a standard deviation. The noise-to-signal ratio, δ, is defined as:

$$\delta = \frac{\text{r.m.s. value of the noise, } w(t)}{\text{r.m.s. value of the signal, } c(t)} \tag{14}$$

The criterion used to evaluate estimation performance is the "estimation error fraction", defined as

$$\varepsilon = \frac{||\varphi - \hat{\varphi}_t||}{||\varphi - \hat{\varphi}_o||} \tag{15}$$

where φ is the true system parameter vector and $\hat{\varphi}_t$ and $\hat{\varphi}_o$ are the estimates at time t and 0. Notice that it is possible in this case to define ε because the true system parameters, given in Table I, are known. The scalar ε represents the normalized vector norm of the estimation error in the parameter space. The initial parameter guesses, $\hat{\varphi}_o$, are taken as zero.

Comparison with Least Squares

Figure 3 shows the estimation error fraction for the LS estimator and the IV estimator for a noise-to-signal ratio of 0.0655. As predicted, the IV estimator outperforms the LS estimator. The improvement is greater the larger the signal to noise ratio, and the two methods are equivalent for $\delta = 0$, i.e., the noise-free system.

Multicollinearity and Selection of Sample Rate

One of the basic assumptions underlying the application of LS or IV estimation to the difference equation model is that the observation matrix X (see Equation 4), which is of order $[(p + q) \times K]$, has rank $(p + q)$. This would not be the case if some or all of the explanatory variables are perfectly correlated, a condition known as multicollinearity [22]. As the degree of correlation between the explanatory variables increases, the moment matrix, $[XZ^T]$, becomes ill-conditioned and approaches singularity.

As the sample rate is increased, correlation between successive values of the output increases, since they become close to each other in time. This causes correlation between the p "output" rows of matrix X, since they are formed by delayed values of the output (see Equation 3). The overall result is a deterioration in estimation performance as the sample rate is increased. On the other hand, a decrease in sample rate results in deterioration in estimator performance due to the loss of dynamic information. These results were confirmed experimentally, as shown in Figure 4, which illustrates the effect of sample rate on the number of samples to convergence for a given estimation error fraction. Fortunately, the optimum sample rate is not a single value but a range which appears as a band on the time spectrum from roughly $T = 0.5$ to $T = 1.25$. In addition, excellent results (less than 0.005 after 250 samples) are still obtained within the band of $T = 0.25$ to $T = 4.0$.

A study was made to determine how the optimum sample-interval band depended upon the ratio of the two time constants of the plant. The results presented in Figure 4 had a time constant ratio of $\tau_2/\tau_1 = 0.5$. The cases were obtained with τ_1 maintained at 2.0 and τ_2 varied to give the selected ratios. The following results are reported for the sample interval range for which ε is less than 0.005 after 250 samples.

Ratio	Range of T
0.25	0.2 to 1.5
0.50	0.25 to 4.0
0.75	0.25 to 4.0
1.00	0.30 to 3.5

The conclusion is that the sample time selection is not sensitive to the time constant ratio. This result is significant because it will facilitate the estimation of a process whose time constants are changing.

Algorithm Initialization

The recursive IV algorithm must be initialized by specification of the initial weighting matrix, P_o, and the initial parameter estimate, $\hat{\varphi}_o$. In his chapter on identification using LS estimation, Lee [17] suggests a mathematical trick: that the computation process be initiated with P_o set to αI where I is the identity matrix and α is large, so that it does not matter what value is used for $\hat{\varphi}_o$ because the algorithm's "memory" is so short that it ignores the initial parameter estimate. Hence, any value may be used and zero is as good as any other. On the other hand, this suggests that if a reasonably good guess can be made of $\hat{\varphi}_o$, then this confidence should be reflected in P_o by lowering the value of α. The effect of noise on the selection of α was studied for IV estimation. The results are shown in Figure 5. These results show that the value of α has a pronounced effect on IV estimations. In fact, if α is chosen to be too high for a given δ, the IV method may not converge at all. However, we found that if α is chosen properly, the results of IV estimation are much improved over the LS method when measurement noise is present.

Parameter Tracking

The dynamic IV algorithm of Equations 11-13 was studied to determine the parameter tracking ability of a nonstationary system. The second-order example of the previous sections was initiated and the gain, k, was instantaneously doubled after 500 samples. The algorithm was therefore required to follow two separate step changes in two of the parameters of the model. The first step exercises the initial "locking" of the algorithm and the second step, initiated at the 500th sample, requires the algorithm to track an abrupt change in two of the plant parameters. The initial estimate of the parameter vector φ was taken to be zero.

The two difference equation parameters affected by the process gain are parameters b_1 and b_2 which

are directly proportional to k. The matrix D of Equation 12 was chosen to be the diagonal matrix

$$D = \begin{bmatrix} 0 & & & \\ & 0 & & \\ & & d & \\ & & & d \end{bmatrix}$$

where d was varied in the study to demonstrate its effect on tracking. With $d = 0$ the dynamic algorithm reduces to the static form of Equation 8 and 9.

Figures 6 and 7 show the excellent tracking performance of the IV method for random measurement noise with a standard deviation, σ_v, equal to 0.0125 which corresponds to a noise-to-signal ratio of 0.0655 for the first 500 samples. The parameter d is indicative of the algorithm's memory as is demonstrated in both figures. In practice, d would be selected experimentally on the basis of desired speed of response and the opposing smoothness. As the level of measurement noise is increased, the IV estimates become proportionately more erratic. This is due to the shorter memory of the dynamic estimator. Results were similar for the case in which all four model parameters were tracked with $D = dI$, as the ratio of the time constants, which affects all four parameters, was changed after 500 samples.

Plant Disturbances

To this point, consideration has been limited to a deterministic system disturbed only by measurement noise. More specifically, w_d in Figure 1 has been considered to be zero and attention has been focused on the w_n term which represents measurement noise. It will be of interest in this section to drop the measurement noise term and examine the single effect of plant disturbances. In this instance, the second-order example is no longer deterministic for it is now considered to be stochastic.

For the case where the plant disturbances are white noise, the estimation problem fulfills all of the assumptions of the LS estimator in the probability limit and that estimator is therefore the optimal estimator. Although the LS estimator is not biased in this case, as less of the output y_t is explained by the explanatory variable u_t, then the variances of the estimates increase and the estimator performance decreases. This point is especially important for closed-loop estimation. The results for the IV estimator in this case are not significantly different.

Figure 8 illustrates the effect of autocorrelated plant disturbances for IV estimation. The autocorrelation factor, s_1, is a measure of the correlation from one sample to the next according to the expression

$$w_t = -s_1 w_{t-1} + v_t \qquad (16)$$

The combination of autocorrelated plant disturbances and lagged values of the input and output cause the LS estimator to become inconsistent [4]. On the other hand, Figure 8 is evidence that the IV estimator is capable of maintaining consistent estimates because it can effectively correct the problem associated with the correlated errors contained in the lagged variables. However, the IV method will not be efficient in this instance because it has dealt only with one of the difficulties and has not corrected the autocorrelation of the disturbance.

Summary

A sequential technique for estimating the parameters of a difference equation model of a process subject to noise has been presented for the open-loop configuration. The technique requires no assumptions to be made on the noise statistics and preserves much of the simplicity of least-squares estimation while dealing effectively with measurement noise.

The method was applied to a deterministic second-order example to illustrate its performance in comparison to LS estimation and to study the selection of various parameters to be used in its implementation. Demonstration was given on the parameter tracking capability of the dynamic form of the IV estimator. The dynamic algorithm is important for use in an adaptive control strategy.

Acknowledgement

This work was sponsored by the Air Force Office of Scientific Research, Air Force Systems Command, USAF under contract 74-2580.

Notation

a	model output parameters
b	model input parameters
c	true system output
d	element of lower bound matrix
D	(nxn) lower bound matrix
E	expected value operator
I	identity matrix
K	total number of observations
k	gain
M	dead time
N	auxiliary model update delay
n	system order (p + q)
p	number of output parameters
P	(nxn) weighting matrix
q	number of input parameters
q_{t-1}	(1xn) random disturbance vector
s_1	autocorrelation factor
T	sample interval
t	time index
u_t	control variable
v_t	white noise
w_t	combined or theoretical disturbance term
W_K	(1xK) vector of disturbances
x_t	(nx1) vector of passed inputs and outputs; exogenous variable
X_K	(nxK) observation matrix
y_t	disturbance-corrupted output; endogenous variable
Y_K	(1xK) vector of outputs
Z_K	(nxK) instrumental-variable matrix
z_t	(nx1) instrumental variable vector
α	initial diagonal element of P_o
δ	noise-to-signal ratio; Kronecker delta function
ε	estimation error fraction
ξ_t	auxiliary model output
σ	standard deviation
τ	time constant
φ	(1xn) vector of model parameters

Literature Cited

1. Smith, C. L., Digital Computer Process Control, Intext, Scranton, Penn., 1972.

2. Young, P. C., "Regression Analysis and Process Parameter Estimation: ... a Cautionary Message", Simulation, Vol. 10, No. 3, March 1968.

3. Kendall, M. G. and A. Stuart, The Advanced Theory of Statistics, Vol. 2, Griffin, London, 1961.

4. Johnston, J., Econometric Methods, 2nd Ed., McGraw-Hill, New York, 1972.

5. Hastings-James, R. and M. W. Sage, "Recursive Generalized-Least-Squares Procedure for Online Identification of Process Parameters", Proceedings, Institution of Electrical Engineers, Vol. 116, No. 12, Dec., 1969, pp. 2057-2062.

6. Isermann, R., U. Baur, W. Bamberger, P. Kneppo and H. Siebert, "Comparison of Six On-Line Identification and Parameter Estimation Methods", Automatica, Vol. 10, No. 1, Jan. 1974, pp. 81-103.

7. Saridis, G. N., "Comparison of Six On-Line Identification Algorithms", Automatica, Vol. 10, No. 1, Jan. 1974, pp. 69-79.

8. Reiersol, O., "Confluence Analysis by Means of Lag Moments and Other Methods of Confluence Analysis", Econometrica, Vol. 9, No. 1, 1941, pp. 1-23.

9. Durbin, J., "Errors in Variables", Review of Int. Statist. Inst., Vol. 22, No. 23, 1954.

10. Joseph, P. J., Lewis and J. Tou, "Plant Identification in the Presence of Disturbances and Application to Digital Adaptive Systems", Transactions AIEE, Vol. 80, 1961, pp. 18-24.

11. Andeen, R. E. and P. P. Shipley, "Digital Adaptive Flight Control System for Aerospace Vehicles", AIAA Journal, Vol. 1, No. 5, May 1963, pp. 1105-1109.

12. Levadi, V. S., "Parameter Estimation of Linear Systems in the Presence of Noise", Presented at the 1964 International Conference on Microwaves, Circuit Theory, and Information Theory, September 7-11, Tokyo, Japan.

13. Wong, K. Y. and E. Polak, "Identification of Linear Discrete Time Systems Using the Instrumental Variable Method", IEEE Transactions on Automatic Control, Vol. AC-12, No. 6, Dec. 1967.

14. Young, P. C., "An Instrumental Variable Method for Real-Time Identification of a Noisy Process", Automatica, Vol 6, 1970, pp. 271-287.

15. Rowe, I. H., "A Boot Strap Method for the Statistical Estimation of Model Parameters", International Journal of Control, Vol. 12, No. 5, 1970, pp. 721-738.

16. Young, P. C., "Applying Parameter Estimation to Dynamic Systems - Part I", Control Engineering, Vol. 16, Oct. 1969.

17. Lee, R. C. K., Optimal Estimation, Identification, and Control, Research Monograph No. 28, M.I.T. Press, Cambridge, Mass., 1964.

18. Kalman, R. E., "A New Approach to Linear Filtering and Prediction Problems", Transactions of the ASME, J. Basic Engr., Vol. 82, March 1960, pp. 35-46.

19. Kalman, R. E. and R. S. Bucy, "New Results in Linear Filtering and Prediction Theory", Transactions ASME, J. of Basic Engr. Series D, Vol. 83, March 1961, pp. 95-108.

20. Eveleigh, V. W., Adaptive Control and Optimization Techniques, McGraw-Hill, New York, 1967.

21. Davies, W. D. T., System Identification for Self-Adaptive Control, Wiley-Interscience, New York, 1970.

22. Klein, L. R., A Textbook of Econometrics, 2nd Ed., Prentice-Hall, Englewood Cliffs, N.J., 1974.

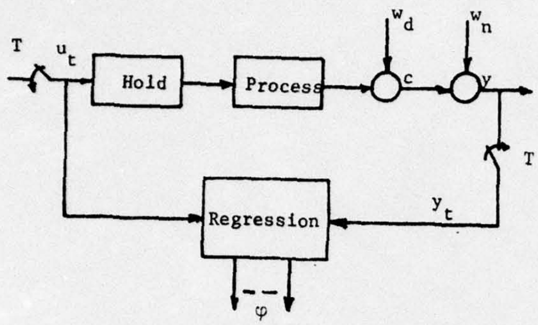
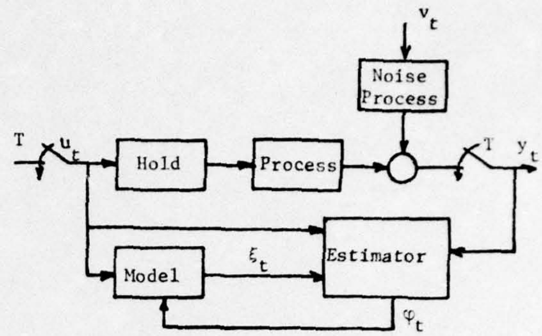


Figure 1. Identification Scheme



$$\xi_t = \hat{\varphi}_{t-1} z_{t-1}$$

where $z_{t-1}^T = [\xi_{t-1} \cdots \xi_{t-p} u_{t-M-1} \cdots u_{t-M-q}]$

Figure 2. Instrumental Variable Estimator

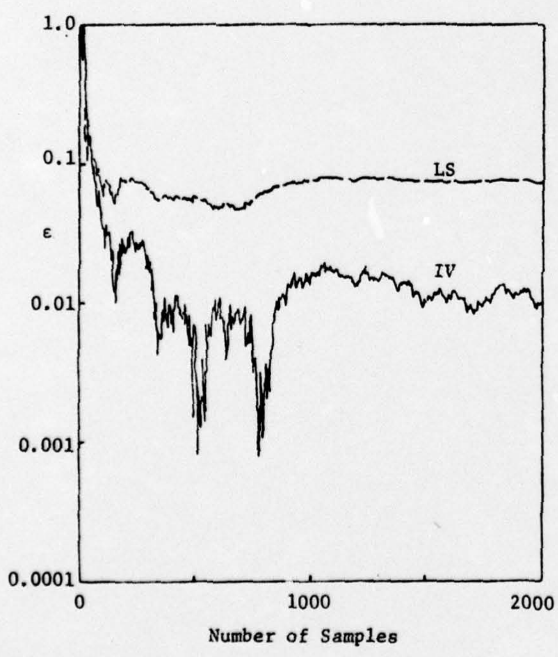


Figure 3. Comparison of LS and IV estimation with $\delta = 0.521$.

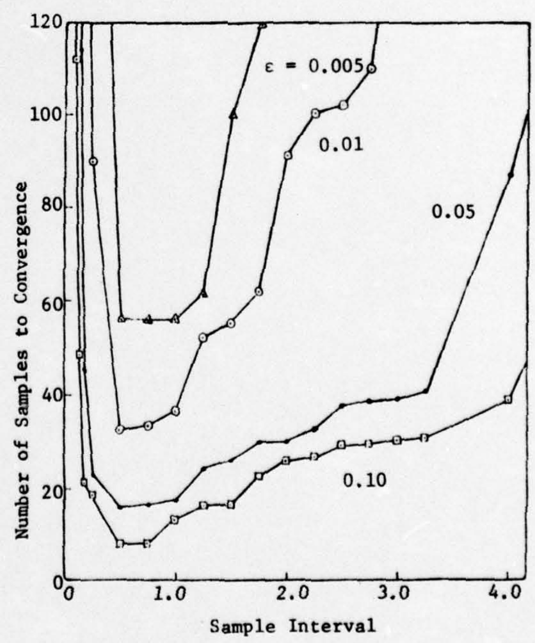


Figure 4. Effect of the sample rate on speed of convergence

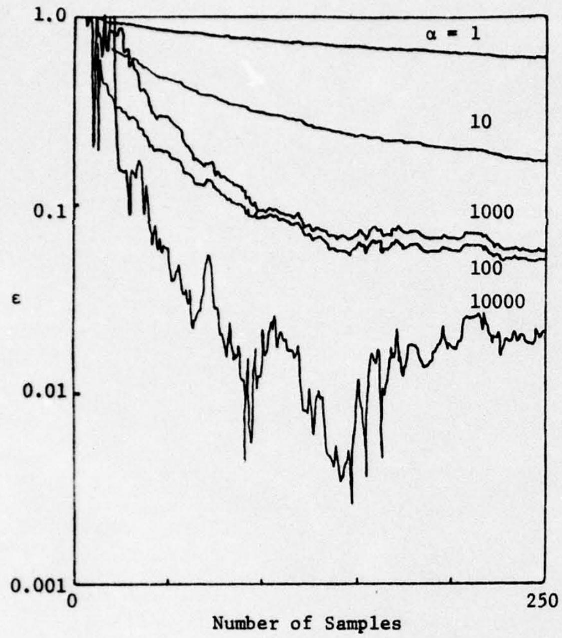


Figure 5. Effect of the variation of α on IV estimation with uncorrelated measurement noise, δ = 0.059.

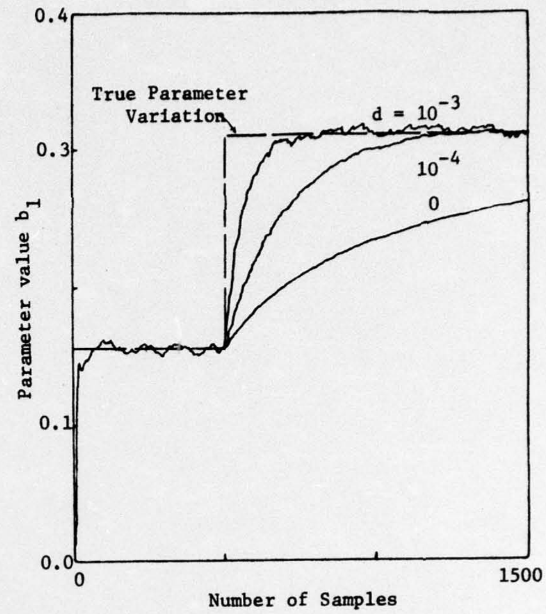


Figure 6. IV tracking estimates of parameter b_1 for selected values of d with $\sigma_v = 0.0125$

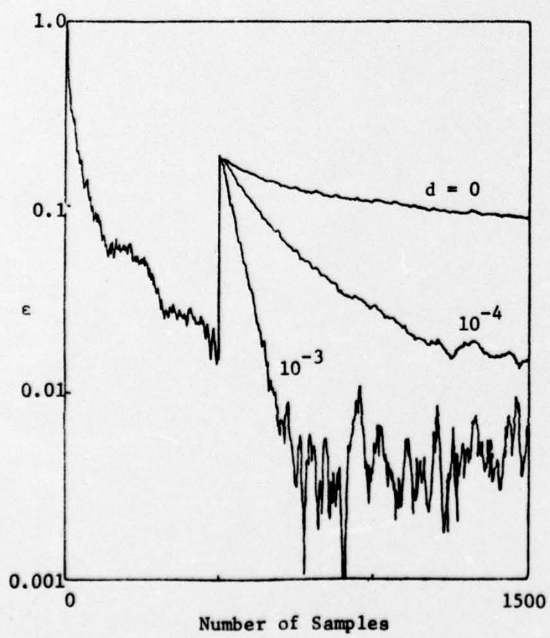


Figure 7. Effect of the variation of d on IV parameter tracking with $\sigma_v = 0.0125$

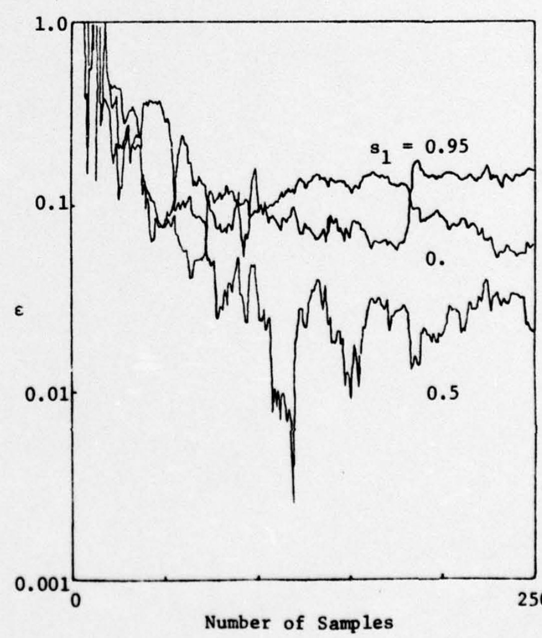


Figure 8. Effect of correlation of plant disturbances on IV estimation with $\sigma_{vp} = 0.025$

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER: AFOSR - TR - 76 - 1171

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle): PROCESS IDENTIFICATION UTILIZING A SEQUENTIAL INSTRUMENTAL VARIABLE REGRESSION ALGORITHM

5. TYPE OF REPORT & PERIOD COVERED: Interim rept.

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s): A. T. Touchstone [redacted] A. B. Corripio

8. CONTRACT OR GRANT NUMBER(s): AF-AFOSR-2580-74

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Louisiana State University
Department of Chemical Engineering
Baton Rouge, Louisiana 70803

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: 61102F; 9769-01

11. CONTROLLING OFFICE NAME AND ADDRESS
Air Force Office of Scientific Research/NM
Bolling AFB, Washington, DC 20332

12. REPORT DATE: 1976

13. NUMBER OF PAGES: 9

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

10 p.

15. SECURITY CLASS. (of this report): UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Identification
Difference equations
Estimation
Instrumental Variable

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Instrumental variable (IV) regression is applied to the estimation of the parameters of a difference equation model of a process subject to noise. The technique preserves the simplicity of least-squares estimation, and is shown to significantly reduce the bias on the parameter estimates caused by measurement noise. A second-order example is used to illustrate the performance of the IV estimator, and to study the selection of sample time and initialization parameters. Demonstration is given on the parameter tracking capability of the dynamic form of the algorithm. The dynamic algorithm is important for use in adaptive control.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

404251

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)